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A General Adaptive Control Structure with a Missile Application

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Abstract

A general adaptive control structure is described which includes, as a special case, conventional model reference adaptive controllers. The relationship between controller parameter errors and measured signal errors are described in a form suitable for application of many parameter estimation techniques. A simplified missile pitch-axis example is given to illustrate the application of the general structure.

1. INTRODUCTION

This paper focuses on the structural aspects of adaptive controllers, omitting a treatment of parameter identification laws. An adaptive control structure representation is developed, which describes, as a special case, conventional model reference adaptive controllers.

The structure includes the combined effect of three control components: (1) a parameterized family of plants, (3) a controller parameterization, and (3) a design rule. By "design rule" we mean the objective of the adaptive system, described as a functional dependence of the tuned controller parameters on the plant parameters.

This paper provides a unified framework for describing a broad class of adaptive controller structures; a "general adaptive control structure" is given which includes, as a special case, conventional model reference adaptive control structures. However, the general structure need not be specialized to be applied; certain "error equations" describing the interrelationship of parameter errors and signal errors in a form suitable for estimation, may be developed directly in terms of the general structure.

Ultimately, it is hoped that the added flexibility afforded by the generalized structure will allow a formal development of robust adaptive control laws for practical systems. As part of an exploration of practical potential, a popular engineering approach to missile autopilot design is re-developed within the formal framework of this paper, and is simulated.

The paper is organized as follows. Section 2 describes the general structure for the case of the tuned (post-adaptation) system. Section 3 describes the untuned case, and develops the error equations in the general framework. Section 4 provides a simple parameter adjustment law, without much analysis, simply to complete the system and allow an example to be given. Section 5 provides the missile autopilot example. Conclusions are given in Section 6.

Notation

For a polynomial

$$G(s) = g_n s^n + g_{n-1}s^{n-1} + \cdots + g_0, \tag{1}$$

the underscore will denote the vector of coefficients, that is,

$$\underline{G} = \begin{bmatrix} g_n \\ g_{n-1} \\ \vdots \\ g_0 \end{bmatrix}. \tag{2}$$

When a polynomial coefficient vector such as $\underline{G}$ is used in an equation requiring a vector of larger dimension, $\underline{G}$ should be understood to include additional zero coefficients corresponding to higher powers of s:

$$G(s) = 0s^{n+k} + \cdots + 0s^{n+1} + g_n s^n + + \cdots + g_0. \tag{3}$$

Similarly, the symbol $\underline{0}$ will denote a zero vector of whatever dimension is appropriate.

An overbar will denote the Toeplitz matrix

$$\overline{G} = \begin{bmatrix} \underline{G} & 0 & 0 \\ 0 & \underline{G} & 0 \\ 0 & 0 \ldots & \vdots \\ \vdots & \vdots & 0 \\ 0 & 0 & \underline{G} \end{bmatrix}. \tag{4}$$

The number of columns will be determined by the context in which the matrix appears.

I will denote an identity matrix of the appropriate dimension.

2. TUNED ADAPTIVE CONTROL SYSTEM

2.1 General Structure

Let u_P and y_P denote, respectively, the input and output of an n^{th} order SISO plant. Let r denote an exogenous command input. Let $\Lambda(s)$ be a chosen Hurwitz polynomial of degree $N \geq n$ and define

$$F_0 := \frac{\begin{bmatrix} s^N \\ s^{N-1} \\ \vdots \\ 1 \end{bmatrix}}{\Lambda(s)} \quad F := \begin{bmatrix} F_0 & \underline{0} & \underline{0} \\ \underline{0} & F_0 & \underline{0} \\ \underline{0} & \underline{0} & F_0 \end{bmatrix} \tag{5}$$

$$w_u := F_0 u_P, \quad w_y := F_0 y_P, \quad w_r := F_0 r \tag{6}$$

$$w := \begin{bmatrix} w_u \\ w_y \\ w_r \end{bmatrix} = F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} \tag{7}$$

2.1.1 Plant

A relationship exists between the elements of $w = [w_u \; w_y \; w_r]^T$ due to the plant input-output relationship.

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Nominally,

$$y_P = \left[\frac{N_P}{D_P}\right] u_P, \tag{8}$$

where N_P and D_P are polynomials. Hence

$$D_P y_P = N_P u_P \tag{9}$$

$$\frac{D_P}{\Lambda} y_P - \frac{N_P}{\Lambda} u_P = 0. \tag{10}$$

With the definition of w above, (10) is

$$\Theta_P^T C_P^T w = 0 \tag{11a}$$

$$\Theta_P^T := [\ -\underline{N}_P^T \mid \underline{D}_P^T\ ] \tag{11b}$$

$$C_P^T := \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}. \tag{11c}$$

Of course, scaling of Θ_P in (11) does not change the right-hand-side of (11a), so we may select any one nonzero parameter in Θ_P at will. This corresponds to assuming, for example, that the plant denominator polynomial D_P is monic.

Allowing all but one parameter to be unknown corresponds to complete pole and zero uncertainty. This high degree of uncertainty is frequently conservative; plant uncertainty often admits a lower-degree parameterization. For this reason, we introduce the parameterization illustrated in Figure 1. In the figure,

$$M(s) = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} \tag{12}$$

is a known stable, proper, linear time-invariant transfer function matrix. The parameter vector $\hat{\Theta}_P$ in this figure can have fewer elements than the corresponding vector in (11).

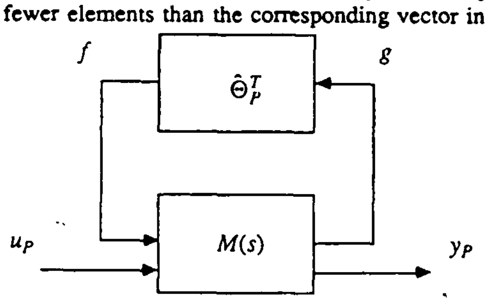

Figure 1
Plant Parameterization

Lemma 1 : Without loss of generality, we may assume that M_{21}^{-1} is stable and proper. (Proof in appendix.)

To relate the structure of Figure 1 to the simple linear constraint of equation (11), we first define

$$w' = \begin{bmatrix} f \\ g \end{bmatrix} \tag{13}$$

where f and g are the signals appearing in Figure 1. Then with the definition

$$\Theta_P = \begin{bmatrix} -1 \\ \hat{\Theta}_P \end{bmatrix}, \tag{14}$$

the relationship $f = \hat{\Theta}_P^T g$ in the figure is equivalent to the equation

$$\Theta_P^T w' = 0. \tag{15}$$

To complete the explanation of the new structure, we relate the signal vector w' to the constructable signal vector w. Through some algebraic manipulation, we get

$$w' = \begin{bmatrix} -M_{21}^{-1} M_{22} & M_{21}^{-1} \\ M_{12} - M_{11} M_{21}^{-1} M_{22} & M_{11} M_{21}^{-1} \end{bmatrix} \begin{bmatrix} u_P \\ y_P \end{bmatrix}. \tag{16}$$

Extracting a common denominator, Λ, yields

$$w' = \begin{bmatrix} \frac{N_{11}}{\Lambda} & \frac{N_{12}}{\Lambda} \\ \frac{N_{21}}{\Lambda} & \frac{N_{22}}{\Lambda} \end{bmatrix} \begin{bmatrix} u_P \\ y_P \end{bmatrix}. \tag{17}$$

Next, letting C_N be the matrix of coefficients of the numerator polynomials, i.e.,

$$C_N^T = \begin{bmatrix} \underline{N}_{11}^T & \underline{N}_{12}^T \\ \underline{N}_{21}^T & \underline{N}_{22}^T \end{bmatrix}, \tag{18}$$

yields

$$w' = C_N^T \begin{bmatrix} w_u \\ w_y \end{bmatrix}. \tag{19}$$

Finally, to obtain precisely the form of (11), simply augment C_N^T with some zero columns to multiply the w_r part of w:

$$C_P^T := [\ C_N^T\ \ \underline{0}\ ] \tag{20}$$

so that (19) becomes

$$w' = C_P^T w \tag{21}$$

and (15) becomes

$$\Theta_P^T C_P^T w = 0. \tag{22}$$

Equation (22) is the generalized version of (11). The dimension of the uncertain vector Θ_P is reduced by the presence of the known matrix C_P, which is calculated from the interconnection structure M. (Incidentally, this interconnection structure applies in the same fashion for multi-input multi-ouput (MIMO) systems, provided M_{21} is left-invertible.)

The plant parameterization of Figure 1 includes, as a special case, the complete parameter uncertainty of (11), by choosing

$$M_{11} = \begin{bmatrix} \underline{0} \\ F_0 \end{bmatrix},\ M_{12} = \begin{bmatrix} F_0 \\ \underline{0} \end{bmatrix},\ M_{21} = 1,\ M_{22} = 0. \tag{23}$$

2.1.2 Control

A broad class of control laws can be constructed by letting u_P be a linear combination of the elements of w as shown in Figure 2. That is,

$$u_P = \psi^T w \tag{24}$$

for some constant vector ψ. Since

$$u_P = \frac{\Lambda}{\Lambda} u_P = \Delta^T w_u, \tag{25}$$

equation (24) is equivalent to

$$\left\{\psi^T - [\Delta^T \mid \underline{0}^T \mid \underline{0}^T]\right\} w = 0. \tag{26}$$

Letting $\Theta_C := \left[\psi^T - [\Delta^T \mid \underline{0}^T \mid \underline{0}^T]\right]$, (26) becomes

$$\Theta_C^T w = 0. \tag{27}$$

In adaptive control, it is important to distinguish between fixed controller parameters and adjustable controller parameters. Thus while (27) describes a very broad class of controllers, one may wish to constrain the tuned controller to some subset. Consequently, we shall study controllers of the form

$$\Theta_C^T C_C^T w = 0, \tag{28}$$

where C_C is a real matrix defining the structure constraints, and Θ_C is the vector of adjustable parameters. Note that since scaling of Θ_C does not change the right-hand-side, we may again assume knowledge of one nonzero element of Θ_C.

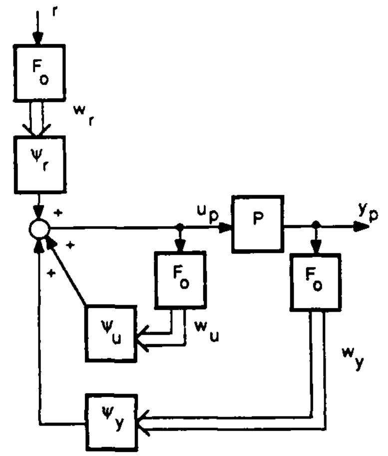

Figure 2
Controller Parameterization

2.1.3 Goal

A broad set of objectives (including model-matching, loopshaping, and pole-placement) can be represented by another linear interrelationship:

$$\Theta_G^T w = 0. \tag{29}$$

2.1.4 Interrelationship

Underlying the notion of "tuned" system is the understanding that the plant equation and the control equation together imply achievement of the goal, without any other assumptions regarding the excitation. This may be stated more precisely.

Consider the scalar field S = { single-input single-output convolution kernels }. Let X be the linear space over S of all triplets $\{u_P, y_P, r\}$ such that each is finite and piecewise continuous.

The plant constraint (22) is

$$\Theta_P^T C_P^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0 \tag{30a}$$

The control constraint (28) is

$$\Theta_C^T C_C^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0. \tag{30b}$$

It is necessary that, together, (30a) and (30b) imply the goal

$$\Theta_G^T F \begin{bmatrix} u_P \\ y_P \\ r \end{bmatrix} = 0 \tag{30c}$$

for all $\{u_P, y_P, r\} \in X$.

Note that (30a) and (30b) are each a linear constraint, which together must imply the third linear constraint (30c). That is, the third constraint lies in the subspace spanned by the first two, hence there must exist scalars $a, b \in S$ such that

$$a\Theta_P^T C_P^T F + b\Theta_C^T C_C^T F = \Theta_G^T F \tag{31}$$

Given fixed values of Θ_P, Θ_G, C_P, C_C, satisfaction of (31) for some (unique) Θ_C and some a, b corresponds to existence (uniqueness) of "tuned system" controller gains. This interrelationship will also have significance later in the discussion of the untuned system.

2.2 Special Case

For added clarity, we give a simple special case of the above general equations. Other interesting examples exist and will be included in a future paper.

Let the plant be $P = \dfrac{N_P(s)}{D_P(s)}$, with N_P Hurwitz, and D_P monic, and of degrees $n-1$ and n respectively (for ease of exposition). Let there be a positive lower bound on the leading coefficient of N_P. Let the coefficents of N_P and D_P be otherwise unknown. Then

$$\Theta_P^T w = 0, \quad \Theta_P^T = [\, -\underline{N}_P^T \mid \underline{D}_P^T \mid \underline{0}^T \,]. \tag{32}$$

Let the goal be

$$y_P = \frac{N_M(s)}{D_M(s)} r \tag{33}$$

where N_M, D_M are of degree $n-1$ and n respectively, with D_M Hurwitz. Thus

$$\Theta_G^T w = 0, \quad \text{with } \Theta_G^T = [\, \underline{0}^T \mid \underline{D}_M^T \mid -\underline{N}_M^T \,]. \tag{34}$$

Let $\Lambda = \Lambda_1\Lambda_2$ where Λ_1 is a degree-one polynomial.

Figure 3 shows a (non-minimal) "tuned" controller which achieves the goal ((33), (34)). Figure 3 is equivalent to

$$\Theta_C^T C_C^T w = 0 \tag{35a}$$

$$C_C = \begin{bmatrix} \overline{\Lambda}_1 & 0 & 0 \\ 0 & \overline{\Lambda}_1 & 0 \\ 0 & 0 & \overline{\Lambda}_1 \end{bmatrix} \tag{35b}$$

$$\Theta_C^T = \left[-\underline{N}_P^T \mid (\underline{D}_P - \underline{D}_M)^T \mid \underline{N}_M^T\right]. \tag{35c}$$

The scalars required to satisfy (31) are

$$a = 1, \quad b = \frac{-1}{\Lambda_1}. \tag{36}$$

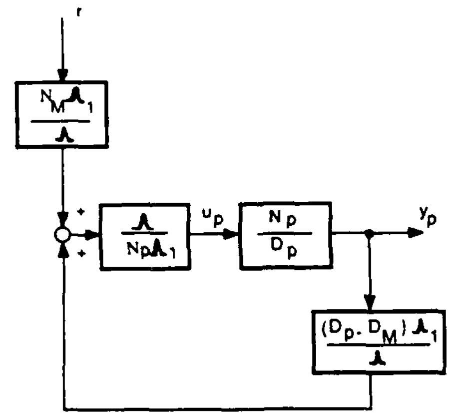

Figure 3
Special Case:
Pole Placement Structure

3. UNTUNED SYSTEM

The untuned-system analysis is based on three sets of equations: (A) the tuned-system equations, (B) the parameter interrelationship, and (C) the untuned-system equations.

(A) Tuned-System Equations

It is now necessary to distinguish between tuned and untuned parameters and signals. Let w^* denote the value of w which would be produced by the tuned system. Thus

$$\Theta_P^T C_P^T w^*(t) = 0 \quad \forall t \tag{37a}$$

$$\Theta_C^T C_C^T w^*(t) = 0 \quad \forall t \tag{37b}$$

$$\Theta_G^T w^*(t) = 0 \quad \forall t \tag{37c}$$

(B) Parameter Interrelationship

Recalling (31), we have

$$a\Theta_P^T C_P^T w + b\Theta_C^T C_C^T w = \Theta_G w \quad \forall\ w \in Fx,\ x \in X. \tag{38}$$

(C) Untuned-System Equations

Let w denote the signal vector which actually arises from the operation of the untuned (and possibly time-varying) system. Regardless of whether or not the system is tuned, the plant input-output equation (22) is satisfied:

$$\Theta_P^T C_P^T w(t) = 0 \quad \forall t. \tag{39a}$$

Let $\theta_C(t)$ denote the controller gains used (in lieu of knowledge of Θ_C). Then

$$\theta_C^T(t) C_C^T w(t) = 0 \quad \forall t \tag{39b}$$

(this is, by definition, the operation of the controller). The goal need not be satisfied, so we define (and construct) an error signal $e(t)$ by

$$\Theta_G^T w(t) =: e(t) \quad \forall t. \tag{39c}$$

Define the controller parameter error vector to be

$$\phi_C(t) = \theta_C(t) - \Theta_C \tag{40}$$

Then the following relationships exist between $e(t)$ and $\phi_C(t)$ (proved in the appendix):

Theorem 1 :

$$e(t) = -b\phi_C^T(t) C_C^T w(t).$$

Theorem 2 : For any chosen LTI filter f,

$$e_1 := fe + fb(\theta_C^T C_C^T w) - \theta_C^T(fbC_C^T w) = \phi_C^T C_C^T(-fbw).$$

The last equation shows that the parameter error ϕ_C is related to the error signal e_1 in an especially simple way: the error signal is the product of the parameter error and a known (that is, easily constructed) signal vector. This simple form is ideally suited for common parameter estimation schemes.

Let us now simplify notation somewhat. Since, as mentioned previously, at least one of the controller parameters in Θ_C may be considered known, it follows that at least one element of ϕ_C is zero. Removing the corresponding row of $C_C^T(-fbw)$ would not affect Theorem 2, so let

$$\hat{w} := C_C^T(-fbw) \text{ with certain rows removed,} \tag{41}$$

$$\phi = \phi_C \text{ with known zero elements removed,} \tag{42}$$

such that $e_1(t) = \phi^T(t)\hat{w}(t).$ (43)

4. ADAPTATION

Theorem 2 shows that the general structure provides parameter error information in an especially simple form. A variety of parameter adjustment laws would work well. Here we briefly state one choice. Let

$$\dot{\theta}_C(t) = \dot{\phi}(t) = \frac{-g_1 Q}{tr(G) + \varepsilon} \tag{44}$$

subject to known upper and lower bounds, and

$$\dot{Q}(t) = -2\sigma Q + \hat{w}(t)e_1(t) + G(t)\dot{\theta}, \quad Q(0) = 0 \tag{45}$$

$$\dot{G}(t) = -2\sigma G(t) + \hat{w}(t)\hat{w}^T(t), \quad G(0) = = 0 \tag{46}$$

where σ, g_1, and ε are any chosen positive gains, and $tr(G)$ denotes the trace of $G(t)$.

Remarks: (1) g_1 controls the rate of parameter adjustment. (2) σ controls the rate of "forgetting" of past information, which is important for time-varying systems. (3) ε is simply a small number to avoid division by zero.

The adjustment laws above are desirable because of the following properties.

Theorem 3 : With the adjustment laws (44) through (46),

$$\dot{\phi}(t) = -g_1 \frac{G(t)}{tr(G) + \varepsilon}\phi(t).$$

Corollary: $\frac{d}{dt}\|\phi\|_2 \le 0$. *(Proof in appendix.)*

A further discussion of adaptation is outside the scope of this paper. For a discussion of adaptive laws resembling those above, the interested reader can see [1], [2].

5. MISSILE EXAMPLE

5.1 Linear Pitch-Axis Model

For the purpose of demonstrating the application of the general adaptive structure described above, we use the following as a "truth model" of the plant.

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta \tag{47a}$$

$$\begin{bmatrix} N_z \\ q \end{bmatrix} = \begin{bmatrix} -Z_\alpha V & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -Z_\delta V \\ 0 \end{bmatrix} \delta \tag{47b}$$

This model describes the short period pitch axis dynamics of an idealized missile. Variable δ is the deflection of the pitch fins, α is the angle of attack, q is the pitch rate, Nz is the normal acceleration, V is the missile velocity, and the coefficients of the state space description (ean##me1:) are given by

$$Z_\alpha = \frac{-\overline{q}S}{mV}C_{N\alpha}, \quad Z_\delta = \frac{-\overline{q}S}{mV}C_{N\delta}$$

$$M_\alpha = \frac{\overline{q}S\overline{C}}{I_{yy}}C_{M\alpha}, \quad M_\delta = \frac{\overline{q}S\overline{C}}{I_{yy}}C_{M\delta}$$

$$C_{N\alpha} = 52, \quad C_{M\alpha} = -63$$

$$C_{N\delta} = 19 - 9.7M + 1.43M^2, \quad C_{M\delta} = -119 + 57.9M - 8.7M^2$$

$$S = 0.31\,ft^2, \quad \overline{C} = 0.625\,ft, \quad m = 7\ slugs, \quad I_{yy} = 71\ slug\text{-}ft^2$$

$$\overline{q} = \tfrac{1}{2}\rho V^2, \quad V = 1100M, \quad \rho = .0024e^{-.0000435h}$$

Specification of mach number M and altitude h determines all of the coefficients. For our purposes, the above model is valid from from mach 0.5 to 4.0, and altitude 0 to 100 kft.

A second order actuator is also included (not shown in (47)), with $\zeta = 0.65$ and $\omega = 250$ rad/s.

It is assumed that Nz and q are measurable via an accelerometer and rate gyro.

5.2 Gain Schedule

Reasonable performance requirements and robustness margins are described in [3] and [4]. Miss distance can be made small when Nz tracks a command Nc with a response time constant between 0.1 sec and 0.4 sec. Significant phase uncertainty occurs above 100 rad/sec, due to actuator and sensor dynamics, and structural modes.

The following gain schedule achieves the performance objectives for dynamic pressure $\overline{q}$ ranging from about 100 to about 10,000.

$$\delta = \frac{60}{M_\delta}(q_c - q) \tag{48}$$

$$\hat{q} = -M_\delta/.093 = \overline{q} \tag{49}$$

$$q_c = \left[\frac{s + .004\hat{q}}{s}\right]\left[\frac{.07}{s + \sqrt{\hat{q}}}\right](Nc - Nz) + \text{dither} \tag{50}$$

Remark: the "dither" signal is not actually a part of the gain schedule; it is a part of the adaptive controller described below.

5.3 Adaptive Control

A few informal observations are in order. The open loop plant described above is a highly resonant second order system. The input-output response of the plant is a strong function of dynamic pressure $\overline{q}$, and a weak function of velocity V. At mid-to-high frequencies ($\approx$ 60 rad/sec), the q/δ transfer function is closely approximated by

$$q = \frac{M_\delta}{s}\delta. \tag{51}$$

We have found that this simple model wth one unknown gain is an acceptable plant parameterization for the purpose of adaptive control.

For ease of identifiability, we add a "dither" signal to q_c of

$$\text{dither} = .01\sin(60t). \tag{52}$$

We represent the first portion of the gain schedule (equation (48)) with the goal

$$q = \frac{60}{s + 60}q_c. \tag{53}$$

The above descriptions fit into the special case of the general structure described earlier in the paper (equations (32) through (34)), with $N_P = M_\delta$, $D_P = s$, $N_M = 60$, $D_M = s{+}60$.

The tuned controller takes the form of equations (35), hence

$$\Theta_C^T = [\ -M_\delta \mid -60 \mid 60\ ]. \tag{54}$$

It happens that for this degree-one case, $\Lambda = \Lambda_1$, and hence $C_C^T w$ simplifies to

$$C_C^T w = \begin{bmatrix} \delta \\ q \\ q_c \end{bmatrix}. \tag{55}$$

We construct the error signal as described previously in Section 3. As mentioned in the general exposition, a filter may be incorporated to emphasize a frequency of interest. We emphasize the dither frequency, where (51) is valid, by choosing the filter

$$f = \frac{N_f\Lambda_1}{D_f}, \quad N_f = s, \quad D_f = s^2 + 2(0.2)60s + 60^2 \tag{56}$$

We construct e_1 as defined in Theorem 2. Then the Theorem yields $e_1 = \phi_C^T C_C^T(-fbw)$. Recalling that, for the special case, $b = \frac{-1}{\Lambda_1}$ (equation (36)), we have that $e_1 = \phi_C^T C_C^T(\frac{N_f}{D_f}w)$. Since only the first element of Θ_C is unknown, $\phi_C^T = [\ \phi \mid 0 \mid 0\ ]$ (see (42) for the definition of ϕ). Thus, following equations (41) through (43), and applying (55),

$$\hat{w} = \left[\frac{N_f}{D_f}\right]\delta \tag{57}$$

$$e_1(t) = \phi^T(t)\hat{w}(t). \tag{58}$$

We implement the adaptive laws (44) through (46), using $\sigma = 5$, $g_1 = 10$, $\varepsilon = 10^{-10}$.

5.4 Simulation Results

The above adaptive system was simulated, using the state equations from section 5.1 with an added second order actuator, and the adaptive system described in section 5.3. The remainder of the gain schedule, namely (50), is accomplished by inserting the estimate θ_{C1} of $-M_\delta$ into (49) to obtain $\hat{q}$, which is then used to determine the other gains of (50).

Two typical simulation runs are shown in Figure 4. The command input Nc was chosen to be a step of 100 ft/s^2 at time zero. The system states were all initially zero.

The chosen flight condition is M=4 at 60kft. The correct value of $-M_\delta$ (i.e., the value of Θ_{C1}) is 126, which is shown as a horizontal line in the figure. The controller parameter estimate θ_{C1} is initialized at 1000 for run (1), and 10 for run (2). In both cases the estimate converges to the correct value, and the response of N_z displays a time constant in the desired range of 0.1 to 0.4 seconds. Small oscil-

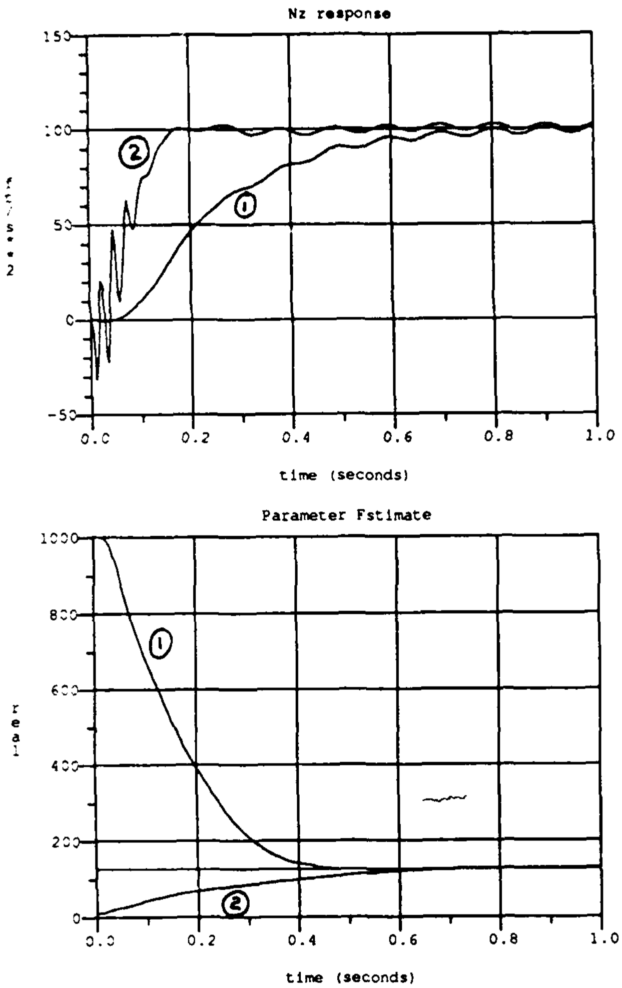

Figure 4
Simulation Results

lations about the commanded value occur due to the use of the dither signal.

A non-adaptive system could not tolerate nearly this degree of $\bar{q}$ uncertainty; it would not even be stable for this range.

5.5 Caveats

The point of this example is to show that the new general adaptive structure and associated theory has application to missile autopilot design. It is not the first application of dithered adaptive control of missiles, nor is it a a final design or a complete theory for missile autopilots. It is, however, an indication that these popular engineering solutions may find more formal support within the theoretical framework of this paper, and future extensions.

A variety of extensions might be considered, such as the use of a 6DOF missile model to allow the dither to take place in the roll axis rather than the pitch axis, or an analysis of the required dither magnitude for acceptable identification in the presence of wind gust, or a study of non-zero initial conditions and time-varying plant parameters.

6. CONCLUSION

A broad class of tuned adaptive control structures can be described by three simple equations:

$$\Theta_P^T C_P^T w(t) = 0 \tag{59a}$$

$$\Theta_C^T C_C^T w(t) = 0 \tag{59b}$$

$$\Theta_G^T w(t) = 0 \tag{59c}$$

These three equations constitute a unified framework for studying a variety of adaptive systems.

A dithered adaptive missile autopilot can be developed within the framework, as indicated by the pitch-axis example of this paper. The notion of an adaptive missile autopilot is not new; our contribution is a formal framework for the development of the adaptive structure. It is hoped that the clarity provided by this framework will expedite the formal analysis of the overall adaptive system.

Appendix

Proof of Lemma 1: One can factor $M_{21}(s)$ into $(H(s)/G(s))\hat{M}_{21}(s)$ where $H(s)$ contains the $\mathbb{C}^+$ zeros of $M_{21}(s)$, $G(s)$ is Hurwitz, G/H is proper, and $\hat{M}_{21}^{-1}$ is stable and proper. Then defining $f_{new} = (H/G)f$, $g_{new} = (H/G)g$, one obtains

$$\begin{bmatrix} g_{new} \\ y_P \end{bmatrix} = \begin{bmatrix} M_{11} & (H/G)M_{12} \\ \hat{M}_{21} & M_{22} \end{bmatrix}, \quad f_{new} = \Theta_P^T g_{new}.$$

□

Proof of Theorem 1: Let $\tilde{w} := w - w^*$ (60)

Then (37a) and (39a) imply $\Theta_P^T C_P^T \tilde{w} = 0.$ (61)

Equation (39b) is equivalent to $\Theta_C^T C_C^T w + \phi_C^T C_C^T w = 0.$ (62)

Equation (37b) and (62) imply $\Theta_C^T C_C^T \tilde{w} + \phi_C^T C_C^T w = 0.$ (63)

Equations (37c) and (39c) imply $e = \Theta_G^T \tilde{w}$ (64)

Equation (38) yields $a\Theta_P^T C_{P_T} \tilde{w} + b\Theta_C^T C_C^T \tilde{w} = \Theta_G^T \tilde{w}.$ (65)

Applying (61) to (65) yields $b\Theta_C^T C_C^T \tilde{w} = \Theta_G^T \tilde{w}.$ (66)

Applying (63), $-b\phi_C^T C_C^T w = \Theta_G^T \tilde{w}.$ (67)

Applying (64) yields $-b\phi_C^T C_C^T w = e.$ (68)

□

Proof of Theorem 2: Letting $\phi_C = \theta_C - \Theta_C$ in the statement of Theorem 1, and noting that the constant Θ_C commutes with b, one finds

$$e = -b(\theta_C^T C_C^T w) + \Theta_C^T (b C_C^T w). \tag{69}$$

Algebraic manipulation yields Theorem 2. □

Proof of Theorem 3: Since the differential equations are locally Lipschitzian, their solutions are unique. One can assume their solutions and verify by differentiation. Thus one may show that

$$G(t) = \int_0^t e^{-2\sigma(t-\tau)} \hat{w}(\tau)\hat{w}^T(\tau)d\tau, \quad Q(t) = G(t)\phi(t) \tag{70}$$

from which the theorem follows. The corollary follows from differentiating $\|\phi(t)\|_2^2$, applying the theorem, and noting that for all symmetric positive semidefinite matrices M, $-\phi^T M \phi \leq 0$. □

References

1. J. M. Krause and P. P. Khargonekar, "On an Identification Problem Arising in Robust Adaptive Control," *Proceedings of the 26th IEEE Conference on Decision and Control*, 1987.
2. G. Kreisselmeier and D. Joos, "Rate of Convergence in Model Reference Adaptive Control," *IEEE Transactions on Automatic Control*, vol. AC-27, June, 1982.
3. F. W. Nesline and M. L. Nesline, "How Autopilot Requirements Constrain the Aerodynamic Design of Homing Missiles," *Proceedings of the 1984 American Control Conference.*
4. F. W. Nesline and M. L. Nesline, "Homing Missile Autopilot Response Sensitivity to Stability Derivative Variations," *Proceedings of the 23rd Conference on Decision and Control*, December, 1984.